Common AIME Geometry Gems

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0 Preface

Some geometry problems show up on a lot of AIME handouts. This is either because they are strong openers, are particularly insightful, or are challenging closers. This handout is a collection of the ones that appear most frequently, so expect this handout to be just a few pages. Since I don't have much taste in 3D Geometry, and they don't show up very frequently on AIME handouts, don't expect to see many.

Complex numbers, if they are not too algebra-reliant, may count (but it's moot, since there aren't many of those problems that appear very often on handouts), but stuff like Roots of Unity do not count.

0.1 References

I am referencing the following Handouts as I make this one:

1. Sean Markan's AIME Syllabus (Jan. 1, 2009)
2. David Altizio's Homemade Problem Collection
3. David Altizio's 100 Geometry Problems (Aug. 30, 2014)
4. Gentle Introduction to AIME
5. Epsilon Summer Series Class 1

I am also referencing my judgment and memory, which is unreliable.

1 Problems

1. On square $ABCD$, point E lies on side $\overline{AD}$ and point F lies on side $\overline{BC}$, so that $BE = EF = FD = 30$. Find the area of square $ABCD$. (2011 AIME-2, Problem 2)

2. Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$? (2004 AMC 10A, Problem 22)

3. Square $AIME$ has sides of length 10 units. Isosceles triangle GEM has base EM, and the area common to triangle GEM and square $AIME$ is 80 square units. Find the length of the altitude to EM in $\triangle GEM$. (2008 AIME, Problem 2)

4. In trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$, let $BC = 1000$ and $AD = 2008$. Let $\angle A = 37°$, $\angle D = 53°$, and M and N be the midpoints of $\overline{BC}$ and $\overline{AD}$, respectively. Find the length MN. (2008 AIME-2, Problem 5)

5. Square $ABCD$ has side length 13, and points E and F are exterior to the square such that $BE = DF = 5$ and $AE = CF = 12$. Find EF^2. (2007 AIME-2, Problem 3)

6. Points A, B, C, D, E and F lie, in that order, on $\overline{AF}$, dividing it into five segments, each of length 1. Point G is not on line AF. Point H lies on $\overline{GD}$, and point J lies on $\overline{GF}$. The line segments $\overline{HC}$, $\overline{JE}$, and $\overline{AG}$ are parallel. Find HC/JE. (2002 AMC 10A, Problem 20)

7. One base of a trapezoid is 100 units longer than the other base. The segment that joins the midpoints of the legs divides the trapezoid into two regions whose areas are in the ratio 2 : 3. Let x be the length of the segment joining the legs of the trapezoid that is parallel to the bases and that divides the trapezoid into two regions of equal area. Find the greatest integer that does not exceed $x^2/100$. (2000 AIME-2, Problem 6)

8. Three circles, each of radius 3, are drawn with centers at $(14, 92)$, $(17, 76)$, and $(19, 84)$. A line passing through $(17, 76)$ is such that the total area of the parts of the three circles to one side of the line is equal to the total area of the parts of the three circles to the other side of it. What is the absolute value of the slope of this line? (1984 AIME, Problem 6)

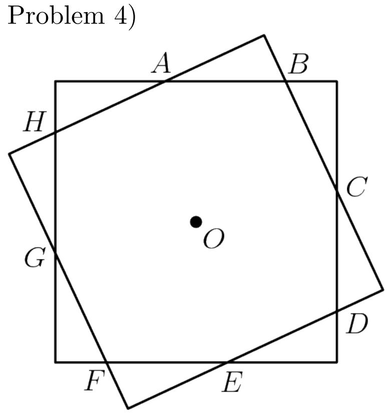
9. In tetrahedron $ABCD$, edge AB has length 3 cm. The area of face ABC is 15cm^2 and the area of face ABD is $12\ \text{cm}^2$. These two faces meet each other at a $30°$ angle. Find the volume of the tetrahedron in cm^3.

2 Other AIME Geometry Gems

1. In a circle, parallel chords of lengths 2, 3, and 4 determine central angles of α, β and $\alpha + \beta$ radians, respectively, where $\alpha + \beta < \pi$. If $\cos \alpha$, which is a positive rational number, is expressed as a fraction in lowest terms, what is the sum of its numerator and denominator? (1985 AIME, Problem 9)

2. In triangle ABC, $\tan \angle CAB = 22/7$, and the altitude from A divides BC into segments of length 3 and 17. What is the area of triangle ABC?

3. The two squares shown share the same center O and have sides of length 1. The length of $\overline{AB}$ is $43/99$ and the area of octagon $ABCDEFGH$ is m/n, where m and n are relatively prime positive integers. Find $m+n$. (1999 AIME, Problem 4)



4. Triangle ABC is isosceles with $AC = BC$ and $\angle ACB = 106^\circ$. Point M is in the interior of the triangle so that $\angle MAC = 7^\circ$ and $\angle MCA = 23^\circ$. Find the number of degrees in $\angle CMB$. (2003 AIME, Problem 10)