Goal: 37
Total: 46

1. Consider $\triangle ABC$ with $\overline{BC}, \overline{AC}, \overline{AB}$ denoted as $a, b, c,$ respectively. If $\angle A = 45^\circ$ and $\angle B = 15^\circ,$ find $\frac{a}{c}.$ (1)

2. Prove that $\cos(3x) = 4\cos^3 x - 3\cos x.$ (2)

3. Find $\frac{\tan 15^\circ + \tan 45^\circ}{\cot 15^\circ + \cot 45^\circ}.$ (2)

4. Find $\tan 0^\circ + \tan 1 + ... + \tan 179^\circ.$ (2)

5. Given that $\sin^{-1} x = y,$ find $\tan y$ in terms of $x.$ (2)

6. Find $\csc 1^\circ \sec 1^\circ + \csc 2^\circ \sec 2^\circ + ... + \csc 359^\circ \sec 359^\circ.$ (3)

7. If $\tan^{-1} x + \tan^{-1} y$ cannot be expressed as $\tan^{-1} z$ for some $z,$ find $xy.$ (3)

8. Given that $\tan x + \tan y = 7$ and $\tan(x + y) = -\frac{7}{9},$ find $\tan x - \tan y,$ provided that $\tan x > \tan y.$ (4)

9. If $\cot x = 3$ and $\cot(x - y) + \cot(x + y) = 6,$ find $\tan y.$ (4)

10. Find the minimum value of $\frac{9x^2 \sin^2 x + 4}{x \sin x}$ for $0 < x < \pi.$ (3)

11. If $0^\circ < x < 180^\circ$ and $\cos x + \sin x = \frac{1}{2},$ find $\tan x.$ (4)

12. Find $\tan x$ if $\frac{\sin^2 x}{3} + \frac{\cos^2 x}{7} = \frac{-\sin(2x) + 1}{10}.$ (4)

13. If θ is acute and $\sin \frac{1}{2}\theta = \sqrt{\frac{x-1}{2x}},$ then find $\tan\theta$ in terms of $x.$ ($\star$ 5)

14. Compute, in degrees, the minimum positive x such that $8\sin x \cos^5 x - 8\sin^5 x \cos x = 1.$ ($\star$ 7)

15. Find $\frac{\tan 1}{1 + \tan 1} + \frac{\tan 2}{1 + \tan 2} + ... + \frac{\tan 89}{1 + \tan 89}.$ ($\star$ 6)