

## Notation

The terms  $\theta, x, y$  are all expressed in degrees.

## Formulas

### Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1.$$

$$\tan^2(\theta) + 1 = \sec^2(\theta).$$

$$\cot^2(\theta) + 1 = \csc^2(\theta).$$

### Odd/Even Functions

$$\sin(\theta) = -\sin(-\theta).$$

$$\cos(\theta) = \cos(-\theta).$$

$$\tan(\theta) = -\tan(-\theta).$$

### Cofunction Identities

$$\sin(\theta) = \cos(90 - \theta), \text{ and } \cos(\theta) = \sin(90 - \theta).$$

$$\tan(\theta) = \cot(90 - \theta), \text{ and } \cot(\theta) = \tan(90 - \theta).$$

### Periodicity Identities

$$\sin(\theta) = \sin(\theta + 360x)$$

$$\cos(\theta) = \cos(\theta + 360x)$$

$$\tan(\theta) = \tan(\theta + 180x)$$

### Sum/Difference Identities

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y).$$

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y).$$

$$\sin(x + y) = \sin(x) \cos(y) + \sin(y) \cos(x).$$

$$\sin(x - y) = \sin(x) \cos(y) - \sin(y) \cos(x).$$

$$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \cdot \tan(y)}.$$

$$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \cdot \tan(y)}.$$

$$\cot(x + y) = \frac{\cot(x) \cdot \cot(y) - 1}{\cot(x) + \cot(y)}.$$

$$\cot(x - y) = \frac{\cot(x) \cdot \cot(y) + 1}{\cot(y) - \cot(x)}.$$

### Double Angle Identities

$$\sin(2x) = 2 \sin(x) \cos(x).$$

$$\cos(2x) = \cos^2(x) - \sin^2(x).$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}.$$

$$\cot(2x) = \frac{1 - \cot^2(x)}{2 \cot(x)}.$$

### Half Angle Identities

$$\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{2}}.$$

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos(x)}{2}}.$$

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{1 + \cos(x)}} = \frac{\sin(x)}{1 + \cos(x)} = \frac{1 - \cos(x)}{\sin(x)}.$$

### Sum to Product Identities

$$\sin(x) + \sin(y) = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right).$$

$$\sin(x) - \sin(y) = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right).$$

$$\cos(x) + \cos(y) = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right).$$

$$\cos(x) - \cos(y) = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right).$$

$$\tan(x) + \tan(y) = \frac{\sin(x+y)}{\cos(x) \cos(y)}.$$

$$\tan(x) - \tan(y) = \frac{\sin(x-y)}{\cos(x) \cos(y)}.$$

### Product to Sum Identities

$$\sin(x) \sin(y) = \frac{1}{2}(\cos(x-y) - \cos(x+y)).$$

$$\sin(x) \cos(y) = \frac{1}{2}(\sin(x-y) + \sin(x+y)).$$

$$\cos(x) \cos(y) = \frac{1}{2}(\cos(x-y) + \cos(x+y)).$$

$$\tan(x) \tan(y) = \frac{\tan(x) + \tan(y)}{\cot(x) + \cot(y)}.$$

### Mollweide's Formulas

Consider  $\triangle ABC$  with  $\overline{BC}$ ,  $\overline{AC}$ ,  $\overline{AB}$  expressed as  $a$ ,  $b$ ,  $c$ , respectively. Then,

$$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin(C)},$$

$$\text{and } \frac{a-b}{c} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos(C)}.$$