Goal: 19
Total: 28

1. Prove that the point of concurrency of the angle bisectors of a triangle is always inside the triangle. (1)

2. Prove that if the incenter and circumcenter of a triangle are the same point, the triangle must be equilateral. (2)

3. Prove that in $\triangle ABC$ with medians AA', BB', CC' and centroid X, that $[AB'X] = [AC'X] = [BA'X] = [BC'X] = [CA'X] = [CB'X]$. (3)

4. Consider $\triangle ABC$ with $\overline{AB} = 5$, $\overline{BC} = 12$, and $\overline{AC} = 13$. Angle bisector AD and median $\overline{AE}$ is drawn such that B, C, D, E are collinear. Find $[ADE]$. (2)

5. Consider $\triangle ABC$ such that $\overline{AB} = 3, \overline{AC} = 5$. Angle bisector AD exists such that $\overline{AD}^2 = \frac{3[ABC]}{2 \cdot \sin(A)}$. Find $\overline{DB}$. (4)

6. In trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$, let $BC = 1000$ and $AD = 2008$. Let $\angle A = 37^\circ, \angle D = 53^\circ$, and M and N be the midpoints of $\overline{BC}$ and $\overline{AD}$, respectively. Find the length MN. (3)

7. For a given triangle $\triangle ABC$, let H denote its orthocenter and O its circumcenter.
   (a) Prove that $\angle HAB = \angle OAC$. ($\star$ 4)
   (b) Prove that $\angle HAO = |\angle B - \angle C|$. ($\star$ 4)

8. Let H_A, H_B, H_C be the feet of the A, B, C altitudes of acute $\triangle ABC$, respectively. Prove that the orthocenter H of $\triangle ABC$ is the incenter of orthic triangle $\triangle H_A H_B H_C$. ($\star$ 5)