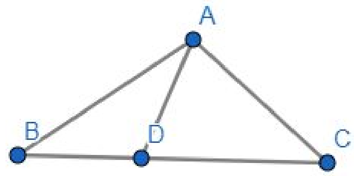
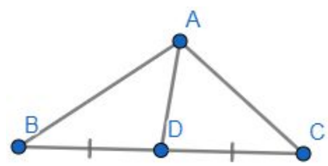
Definition

To define our basic triangle centers we need to define the cevians that make them, and to define specific cevians we must first define a cevian.

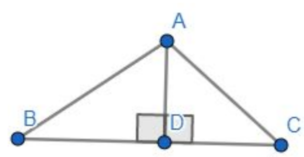
A **cevian** is a line segment with one endpoint as a vertice of a triangle and the other endpoint on the opposite side of the triangle. (Note that this can be the extension of the side!)



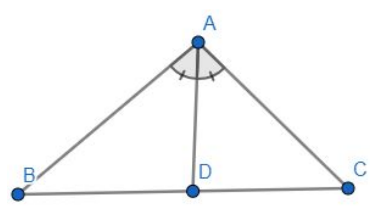
A **median** is a cevian such that $\overline{BD} = \overline{CD}$. They intersect to form the **centroid**.



An **altitude** is a cevian such that $\angle ADB = \angle ADC = 90^\circ$. They intersect to form the **orthocenter**.



An **angle bisector** is a cevian such that $\angle BAD = \angle CAD$. They intersect to form the **incenter**, or the center of the **incircle**, which is the unique circle that can be inscribed within a triangle.



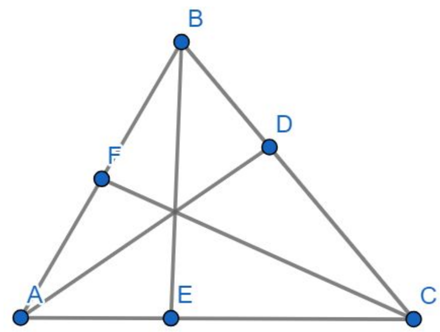
Now we look at **perpendicular bisectors**, which are not necessarily cevians but are incredibly important as well. The perpendicular bisector of $\overline{BC}$ is the locus of point equidistant from $B, C,$ that is, the locus of points X such that $BX = CX$. They intersect to form the **circumcenter**, or the unique point equidistant from the vertices, which is also the center of the **circumcircle** of the triangle, or the unique circle that can be circumscribed around a triangle.

Notation

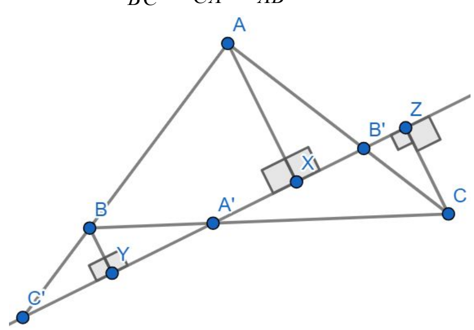
Take $\triangle ABC$ and denote BC, AC, AB as a, b, c and $\angle BAC, \angle ABC, \angle ACB$ as $\angle A, \angle B, \angle C,$ respectively.

Formulas

Ceva's Theorem states that for cevians $AD, BE, CF,$ the cevians are concurrent if and only if $\frac{BD}{CD} \cdot \frac{CE}{AE} \cdot \frac{AF}{BF} = 1.$



Menelaus's Theorem states that for A', B', C' on lines BC, AC, AB respectively (note that these lines can be extended if necessary), A', B', C' are collinear if and only if $\frac{\overline{AC'}}{\overline{BC'}} \cdot \frac{\overline{BA'}}{\overline{CA'}} \cdot \frac{\overline{CB'}}{\overline{AB'}} = 1.$



Stewart's Theorem states that for cevian AD and variables a, b, c, m, n, d representing $\overline{BC}, \overline{AB}, \overline{AC}, \overline{DC}, \overline{BD}, \overline{AD}$ respectively, $man + dad = bmb + cnc.$

The Angle Bisector Theorem states that for $\triangle ABC$ with angle bisector $AD,$ $\frac{\overline{AB}}{\overline{BD}} = \frac{\overline{AC}}{\overline{CD}}.$