Goal: 29
Total: 35

1. Consider a right triangle such that $\sin(\theta) = \frac{3}{5}$. Find $\cos(\theta)$. (1)

2. Prove that $\sin(\theta) = \cos(90 - \theta)$. (1)

3. Prove that $\sin^2(\theta) + \cos^2(\theta) = 1$. (In trigonometry, $\sin^2(\theta) = (\sin(\theta))^2$, not $\sin(\sin(\theta))$. The same is true for cosine.) (1)

4. A right triangle with an angle θ such that $\sin(\theta) = \frac{5}{13}$ has a hypotenuse of 117. Find its area. (2)

5. Prove that $\tan^2(\theta) + \sin^2(\theta) = \tan^2(\theta) \cdot (2 - \sin^2(\theta))$. (2)

6. Consider $\triangle ABC$ with $\overline{BC}, \overline{AC}, \overline{AB}$ denoted as a, b, c, respectively. If $\frac{\tan(\frac{1}{2}[A-B])}{\tan(\frac{1}{2}[A+B])} = \frac{1}{5}$, find $\frac{a}{b}$. (2)

7. Consider $\triangle ABC$ with $\overline{BC}, \overline{AC}, \overline{AB}$ denoted as a, b, c, respectively. If $a = 4$, $b = 2\sqrt{6}$, and $c = 2\sqrt{3} + 2$, find $\angle A, \angle B, \angle C$. (3)

8. Consider $\triangle ABC$ with $\overline{BC}, \overline{AC}, \overline{AB}$ denoted as a, b, c, respectively. If $a = 6$, $b = 4$, and $\angle C = 120^\circ$, find $[ABC]$. (2)

9. Consider $\triangle ABC$ with $\overline{BC} = 5$. Then have $\triangle DEF$ with $\overline{EF} = 10$. If the circumcircle of $\triangle DEF$ has an area four times the area of $\triangle ABC$, then the two values of $\angle D$ are x, y such that $x > y$. If $\frac{x}{y} = 3$, find the area of the circumradius of $\triangle ABC$. (2)

10. If $\sin(x) = \frac{4}{5}$, find $\tan(45 - x)$. (Assume that $0 < x < 90$ for this problem. This problem is written in degrees.) (3)

11. In circle O with radius 6, $arc(AB) = 60^\circ$ and $arc(CD) = 90^\circ$. Find the difference in lengths of segments CD and AB. (3)

12. Prove that $(\csc(\theta) - 1)(\csc(\theta) + 1)(\sec(\theta) - 1)(\sec(\theta) + 1) = 1$, for all θ such that $\csc(\theta)$ and $\sec(\theta)$ are defined. (3)

13. Given triangle $\triangle ABC$ with $\overline{BC}, \overline{AC}, \overline{AB}$ denoted as $a, b, c,$ respectively, find the circumradius of $\triangle ABC$ if $a \cdot \csc(A) = 8.$ (3)

14. Find the minimum value $\csc^2(\theta) + \sec^2(\theta)$ can take. ($\star$ 4)

15. Prove that in $\triangle ABC,$ $\cot(\frac{A}{2}) + \cot(\frac{B}{2}) + \cot(\frac{C}{2}) = \cot(\frac{A}{2}) \cdot \cot(\frac{B}{2}) \cdot \cot(\frac{C}{2}).$ (3)

16. Prove that $\tan^{-1}(x) = \cot^{-1}(\frac{1}{x}).$ (2)