IGP

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Abstract

IGP (Individual to Group Practice) is a new review/practice method designed for mathematics classes. In this paper, the general structure of IGP will be explained, advantages and setbacks will be gone over, and an example of an IGP problem set will be gone over.

1 Prerequisites

A few things to note before you consider using this.

1. You need a relatively small class. 6-10 students.
2. You need to know the students relatively well.
3. The time commitment is insane. Takes 10-12 hours to prepare one of these.
4. Class sessions need to be at minimum 1 hour and 30 minutes long.

2 General

IGP was created on the principle that it is hard to get kids to do homework, especially for review. As thus, it aims to spend as little time as possible to introduce the class to a lot of problems.

The crucial difference is that each student gets 20 minutes to work on a handout; each handout is different.

3 Details

Handout structure looks like this.

1. Individual handouts have 4 or 5 problems based on class size, class time, topics, and size of the problem base.
2. One of these is a warmup that everyone has; this facilitates discussion and is a good starter for solving too.

3. Full handout has all problems on individual handouts plus a couple of problems in database that were considered or are identical in idea.
4. Full handout should be organized by main idea.
5. Individual problems should cite the location they are on the full handout. (This is mostly just for aesthetics.)

Class structure looks roughly like this.

1. 20 minutes to do problems on individual handouts.
2. Give the kids the full handouts; 10 minutes of discussion and letting them glance the full handout.
3. Someone presents warmup in around 5 minutes.
4. Students present some problems they solved.
5. Give 20 to 30 minutes for students to work on big handout together!
6. Teacher presents harder problems, talks about main ideas and overarching themes.

4 Example Database

This is the IGP Database for the 9/18/2018 MPP class.

4.0 Warmups

1. Consider rectangle $ABCD$ with $AB = 6, BC = 8$. Let M be the midpoint of AD and let N be the midpoint of CD. Let BM, BN intersect AC at X, Y. Find XY.
2. Consider $\triangle ABC$ with $AB = 13, BC = 15, CA = 14$. If M is the midpoint of BC and P is a point on AC such that $MP \perp AC$, find MP.

   Variation: Consider $\triangle ABC$ with $AB = 13, BC = 15, CA = 14$. If M is the midpoint of AB and P is a point on AC such that $MP \perp AC$, find MP.

4.1 Area of a Triangle

1. Prove that
$$[ABC] = \frac{a^2 \sin B \sin C}{2 \sin A}.$$
2. If two side lengths of a triangle are given to be 10 and 11, what is the maximum possible area of this triangle?
3. Prove $[ABC] = \frac{abc}{4R}$.

4. Prove $[ABC] = \frac{1}{2}ab\sin C$.
5. Prove $[ABC] = rs$.
6. A triangle has side lengths 4 and 8, and it has an area of $3\sqrt{15}$. Find the possible lengths of the third side.
7. Find the length of the altitude to the 14 inch side of a triangle whose two other sides have lengths of 13 inches and 15 inches.
8. Tangents from point C to circle O are extended to A and B such that AB is tangent to O at X. If the perimeter of $\triangle ABC$ is 50 and $[ABC] = 100$, find the area of circle O.

4.2 Triangle Centers

1. In trapezoid $ABCD$ with $BC||AD$, let $BC = 1000$ and $AD = 2008$. Let $\angle A = 37°, \angle D = 53°$, and M, N be the midpoints of BC and AD respectively. Find the length MN.
2. Consider $\triangle ABC$ with $AB = 5$, $BC = 12$, and $AC = 13$. Angle bisector AD and median AE is drawn such that B, C, D, E are collinear. Find $[ADE]$.
3. The sides of $\triangle BAC$ are in the ratio $2 : 3 : 4$. BD is the angle bisector drawn to the shortest side AC, dividing it into segments AD and CD. If the length of AC is 10, then find the length of the longer segment of AC.
4. If triangle PQR has sides 40, 60, and 80, then the shortest altitude is K times the longest altitude. Find the value of K.

4.3 Telescoping

See my telescoping handout. Any problem on it applies. A few easier ones though:

1. Find $\frac{1}{1\cdot 2} + \frac{2}{2\cdot 4} + \frac{3}{4\cdot 7} + \frac{4}{7\cdot 11} + \frac{5}{11\cdot 16}$.
2. Find
$$\frac{1}{1\cdot(1+2)} + \frac{1}{2\cdot(2+2)} + \cdots + \frac{1}{21\cdot(21+2)}$$
rounded to the nearest integer.
3. Simplify $(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$.
4. Find $\frac{1}{4} + \frac{1}{10} + \frac{1}{18} + \frac{1}{28} + \frac{1}{40} + \frac{1}{54} + \frac{1}{70} + \frac{1}{88} + \frac{1}{108}$.
5. Find $\sum_{n=1}^{13} \frac{1}{t(n)}$, where $t(n) = \sum_{i=1}^{n} i$ ($t(n)$ is the nth triangular number).
6. If $f(x) = \frac{x^2}{x^2-1}$, find $\prod_{n=3}^{50} f(n)$.

4.4 Counting

1. A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

2. Find the probability the product of the bottom face of 3 dice is composite.

3. How many 3 digit numbers have digits that when multiplied out, have an even product?

4. Find the number of subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ that are subsets of neither $\{1, 2, 3, 4, 5\}$ nor $\{4, 5, 6, 7, 8\}$.

5. Brian shoots 6 balls, each making with a probability of p. Find the value of p that maximizes the probability that he makes exactly 3 of 6 shots.

6. How many 4 digit falling numbers are there? (A falling number is a number whose last digit is strictly smaller than its second-to last digit, and so on. Ex. 4321)

4.5 Closing Thoughts

1. This method will not work if you don't have time or have lots of kids.

2. I don't think this precise method will work for English/History classes, but I do think that the general **idea** of not having all kids do the same thing is valid. Specifically, I find that I do not care to hear the same presentation 10 times; I would much rather listen to different presentations from different sources.

3. This should work for USACO/USAPhO/USAChO, but I'm too lazy to figure out the specifics.

4. It is unrealistic and unnecessary to have each class be IGP. IGP should be more of a review.