Goal: 18 points
Total: 25 points

1. Prove the perpendicular bisectors of a triangle are concurrent. (1)

2. Prove the angle bisectors of a triangle are concurrent. (1)

3. Prove the medians of a triangle are concurrent. (2)

4. Prove the altitudes of a triangle are concurrent. (2)

5. Prove that the orthocenter, centroid, and the circumcenter of a triangle are collinear. ($\star$ 4)

6. Consider $\triangle ABC$ with point D on BC. Let M, N be the circumcenters of $\triangle ABD$ and $\triangle ACD$, respectively. Let the circumcircles of $\triangle ACD$ and $\triangle MND$ intersect at $H \neq D$. Prove A, H, M are collinear. ($\star$ 6)

7. In $\triangle ABC$ lines CE and AD are drawn so that $\frac{CD}{DB} = \frac{3}{1}$ and $\frac{AE}{EB} = \frac{3}{2}$. Let $r = \frac{CP}{PE}$ where P is the intersection point of CE and AD. Find r. (3)

8. Let CH be the altitude of acute $\triangle ABC$. The points X, Z, Y lie on lines CA, CH, CB respectively in such a manner that $AX = AC, BY = BX$, and $HZ = HC$. Prove that X, Y, and Z are collinear. (3)

9. Let the incircle of $\triangle ABC$ touch BC, CA, AB at X, Y, Z, respectively. Show that AX, BY, CZ are concurrent. (3)