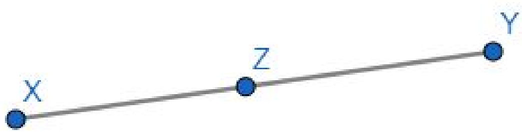
Definition

We say points X, Y, Z are **collinear** if there exists a line that passes all three of them.

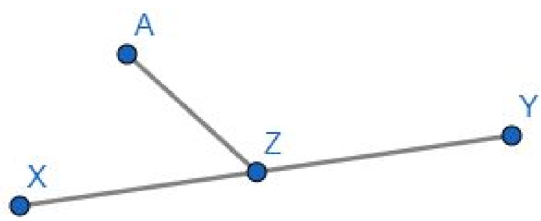
We say lines i, j, k are **concurrent** if there exists a point that all three of them pass.

Formulas

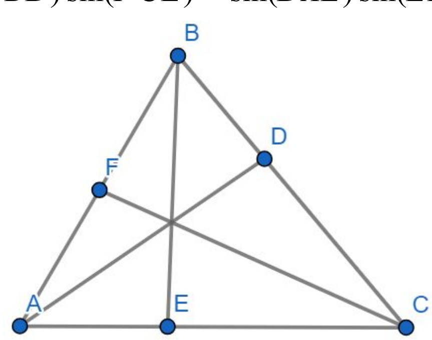
If Z lies on line segment XY, then $XZ + YX = XY$. With the help of directed segments, we generalize it to Z on line XY.



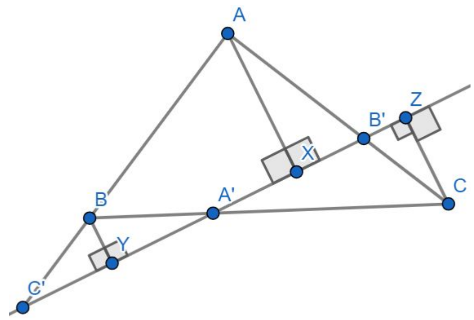
Let Z lie on line segment XY. Points X, Y, Z are collinear if and only if $\angle XZP + \angle YZP = 180^\circ$ for all points P. (This does not necessarily have to be limited to one extra point!)



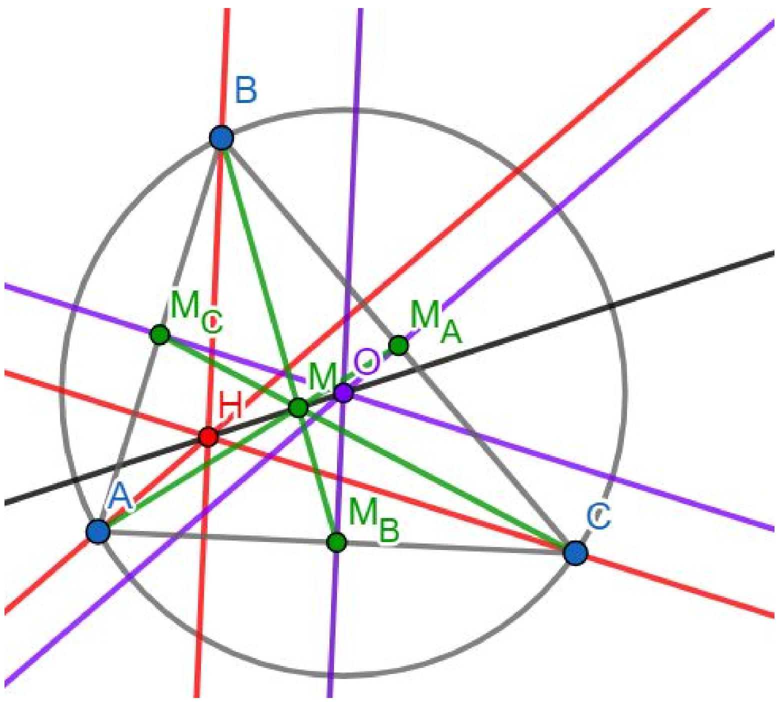
Ceva's Theorem states that for cevians AD, BE, CF, the cevians are concurrent if and only if $\frac{BD}{CD} \cdot \frac{CE}{AE} \cdot \frac{AF}{BF} = 1$. Also,
$\sin(DAF)\sin(EBD)\sin(FCE) = \sin(DAE)\sin(EBF)\sin(FCD)$.



Menelaus's Theorem states that for A', B', C' on lines BC, AC, AB respectively (note that these lines can be extended if necessary), A', B', C' are collinear if and only if $\frac{\overline{AC'}}{\overline{BC'}} \cdot \frac{\overline{BA'}}{\overline{CA'}} \cdot \frac{\overline{CB'}}{\overline{AB'}} = 1.$



The Euler Line passes through the circumcenter, orthocenter, and centroid of any triangle. It is guaranteed to exist. Furthermore, the distance from the centroid to the orthocenter is twice its distance from the circumcenter.



<u>Techniques</u>

Homothety is a tool to prove the collinearity of a group of points. Most famously, it can prove the existence of the Euler Line with some angle chasing. Homothety is based on translating a point respective to another on the line they both lie on.