Goal: 33
Total: 38

1. If $\angle C = 50^\circ$, $\angle B = 60^\circ$, and $\angle A = 70^\circ$, find $arc(AB) - arc(DE)$.  (1)

![Diagram](image1)

2. Find $x$.  (1)

![Diagram](image2)

3. Find $x$.  (1)

![Diagram](image3)

4. Find $x$.  (1)

![Diagram](image4)

5. Given that $A$, $B$, $C$, and $D$ are all on the circumference of the same circle, that $BE$ is the angle bisector of $\angle BAC$, that $\angle AEB = \angle CEB$, and that $\angle ADC = 50^\circ$, find $\angle BAC$.  (2)

![Diagram](image5)
6. Given points $A, B, C, D, E$ such that $BE$ is the angle bisector of $\angle ABC$, $\angle AEB = \angle CEB$, $\angle BAC + \angle BDC = \angle ABD + \angle ACD$, and $\angle ADC = 48^\circ$, find $\angle BCA$. (2)

7. Points $A, B, Q, D$, and $C$ lie on the circle as shown and the measures of arcs $BQ$ and $QD$ are $42^\circ$ and $38^\circ$ respectively. Find $\angle P + \angle Q$. (2)

8. Segments $PA$ and $PT$ are tangent to the circle. Find $\angle TXA$ if $\angle P = 42^\circ$. (2)

9. Consider any cyclic pentagon (a pentagon that can be inscribed within a circle) $ABCDE$. Then prove that, no matter what, $ABCP$ is not cyclic, where $P$ is the center of the circle. (2)

10. Consider chord $AB$ of length 8 inside a circle of radius 5. Prove that only one line $DE$ has a length of 2 such that $D$ is on the arc $AB$ and $E$ is on the line $AB$. (3)

11. Consider points $A, B, I$ such that $\overline{AI} = \overline{BI}$. Given a point $X$ such that $\angle IAX = \angle IBX = 90^\circ$, find $\overline{AX} - \overline{BX}$. (2)

12. Given that $\overline{AD} = 4$, $\overline{DC} = 8$, $\overline{AH} = 1$, and $\overline{EH} = 1$, find the area of $\triangle ABD$. (2)
13. Consider \( \triangle ABC \) with inradius \( r \) such that \( AB = 9 \), \( BC = 12 \), and \( AC = AB + BC - 2r \). Find \([ABC]\). (3)

14. Consider \( \overline{AB} = x \) and circle \( N \) centered at \( B \) with radius \( r \) such that \( r < x \). Find the length of the tangent from \( A \) to \( N \). (3)

15. Given that \( m\angle BAC = m\angle BGC = 40^\circ \), \( m\angle ABG = 80^\circ \), \( m\angle GEB = 2m\angle DBE \), and \( m\angle DBE = m\angle GBE \), find \( m\angle ADB \). (4)

16. Consider \( \triangle ABC \) with point \( D \) on \( BC \). Let \( M, N \) be the circumcenters of \( \triangle ABD \) and \( \triangle ACD \), respectively. Let the circumcircles of \( \triangle ACD \) and \( \triangle MND \) intersect at \( H \neq D \). Prove \( A, H, M \) are collinear. (\( \ast \) 7)