Definitions

A **circle** is the locus of points a set distance away from a certain point, known as the **center**. This set distance is known as the **radius**.

A **chord** is a line segment formed by two distinct points on a circle.

A **secant** is a line that intersects a circle twice.

A **tangent** is a line that intersects a circle once.

The measure of $arc(AB)$ of circle with center I is the measure of $\angle AIB$. Unless specified, this means the minor arc, or the smaller arc.

Formulas

The Inscribed Angle Theorem states that given two chords AB and BC, the measure of $\angle ABC$ is half of $arc(AC)$. In other words, $\angle ABC = \frac{1}{2}arc(AC)$.

Have points A, C be on a circle. The Angle of Intersecting Secants/Tangents Theorem states that given two secants or tangents AB and BC that intersect the circle at points the D, E, $\angle ABC = \frac{1}{2}arc(AC) - \frac{1}{2}arc(DE)$.

Consider chord AB and a tangent line that intersects the circle at C. Without loss of generality, let A be to the left of B, C, and let the tangent line at C be XY, such that X is to the left of C and Y is to the right of C. Then, $\angle ABC = \angle ACX$, and $\angle BAC = \angle BCY$.

Given cyclic quadrilateral $ABCD$, the following properties are true.

$$\angle A + \angle C = \angle B + \angle D = 180^\circ$$
$$\angle ABD = \angle ACD$$
$$\angle BCA = \angle BDA$$
$$\angle BAC = \angle BDC$$
$$\angle CAD = \angle CBD$$

Power of a Point states that given chords AB and CD that intersect at E, $\overline{AE} \cdot \overline{BE} = \overline{CE} \cdot \overline{DE}$.

It also states that given tangent line AB such that A is on the circle and secant BD that intersects the circle again at point $C,$ with D being on the circle, we have

$$\overline{AB}^2 = \overline{BC} \cdot \overline{BD}.$$

Finally, it states that given secants AC and CE where points A, C are on the circle that intersect the circle again B and $D,$ respectively, $\overline{CB} \cdot \overline{CA} = \overline{CD} \cdot \overline{CE}.$

Any line tangent to a circle is perpendicular to the radius that intersects it.