Definitions

A **point** has no length. It denotes a location.

A **line segment** is the shortest path between two points.

A **line** is the infinite extension of two line segments such that any two points on the line form a line segment that lies on the line.

An **angle** is formed by two rays $BA$ and $BC$ that share a common endpoint. The angle is the smallest amount that ray $BA$ needs to be rotated to form ray $BC$. (This means it can be either rotated clockwise or counterclockwise.)

A **plane** is a flat and infinitely extending surface.

Important Properties

The **transitive property** states that if $a = b$ and $b = c$ then $a = c$.

The **midpoint** of line $AB$ is the point $X$ such that $X$ lies on $AB$ and $AX = BX$. There is one unique midpoint for every line.

Points $A, B, C$ are **collinear** if all the points $A, B, C$ can be connected by a single line. In general, points $A_1, A_2...A_n$ are collinear if they all lie on the same line.

Lines $AB, CD, EF$ are **concurrent** lines if there is a common point that lies on all of the lines $AB, CD, EF$. In general, lines $A_1B_1, A_2B_2...A_nB_n$ are concurrent if there is a point that lies on all of these lines.

Lines $AB$ and $CD$ are **coplanar** if they lie on the same plane.

Lines $AB$ and $CD$ are **parallel** if no point lies on $AB$ and $CD$, and if $AB$ and $CD$ are coplanar. This is denoted as $AB \parallel CD$.

Have lines $AB$ and $CD$ intersect at $X$. Lines $AB$ and $CD$ are **perpendicular** if $\angle AXC = 90^\circ$. This is denoted as $AB \perp CD$. 
Then, we shall define some shapes. We begin by defining a **circle** as the locus of points a constant distance, known as the radius, away from a point, known as the center.

![Circle](image)

Then, we define **triangle** \(ABC\) as the plane bounded by the lines \(AB, BC, CA\). We can then define **quadrilateral** \(ABCD\) as the plane bounded by the lines \(AB, BC, CD, DA\).

In general, for \(n \text{-gon } A_1A_2...A_n\), we define it as the plane bounded by the lines \(A_1A_2, A_2A_3...A_{n-1}A_n, A_nA_1\).

We define \(\triangle ABC\) and \(\triangle DEF\) to be **congruent** if \(\overline{AB} = \overline{DE}, \overline{BC} = \overline{EF}, \overline{CA} = \overline{FD}, \angle A = \angle D, \angle B = \angle E,\) and \(\angle C = \angle F\). This is denoted as \(\triangle ABC \cong \triangle DEF\). Note that \(\triangle ABC \cong \triangle DEF\) does not necessarily mean \(\triangle ABC \equiv \triangle DEF\).

In general, \(n\)-gons \(A_1A_2...A_n\) and \(B_1B_2...B_n\) are congruent if \(\overline{A_iA_{i+1}} = \overline{B_iB_{i+1}}\) for \(0 < i < n\), \(\overline{A_nA_1} = \overline{B_nB_1}\), and \(\angle A_j = \angle B_j\) for all \(0 < j < n + 1\). While this book will denote this as \(A_1A_2...A_n \equiv B_1B_2...B_n\), this is not a very common notation and to the best of my knowledge, this notation is unique to *Exploring Euclidean Geometry*.

Additionally, \(\triangle ABC\) is **similar** to \(\triangle DEF\) if there is some constant \(x\) such that \(\overline{AB} = x\overline{DE}, \overline{BC} = x\overline{EF}, \overline{CA} = x\overline{FD}, \angle A = \angle D, \angle B = \angle E,,\) and \(\angle C = \angle F\). This is denoted as \(\triangle ABC \sim \triangle DEF\). Similar to congruence, note that \(\triangle ABC \sim \triangle DEF\) does not necessarily mean \(\triangle ABC \propto \triangle DEF\).

In general, \(n\)-gons \(A_1A_2...A_n\) and \(B_1B_2...B_n\) are similar if there exists a constant \(x\) such that \(A_iA_{i+1} = xB_iB_{i+1}\) for \(0 < i < n\), \(\overline{A_nA_1} = \overline{xB_nB_1},\) and \(\angle A_j = \angle B_j\) for all \(0 < j < n + 1\). This will be denoted as \(A_1A_2...A_n \sim B_1B_2...B_n\).