Definitions

A **point** has no length. It denotes a location.

A **line segment** is the shortest path between two points.

A **line** is the infinite extension of two line segments such that any two points on the line form a line segment that lies on the line.

An **angle** is formed by two rays BA and BC that share a common endpoint. The angle is the smallest amount that ray BA needs to be rotated to form ray BC. (This means it can be either rotated clockwise or counterclockwise.)

A **plane** is a flat and infinitely extending surface.

Important Properties

The **transitive property** states that if $a = b$ and $b = c$ then $a = c$.

The **midpoint** of line AB is the point X such that X lies on AB and $\overline{AX} = \overline{BX}$. There is one unique midpoint for every line.

Points A, B, C are **collinear** if all the points A, B, C can be connected by a single line. In general, points $A_1, A_2 ... A_n$ are collinear if they all lie on the same line.

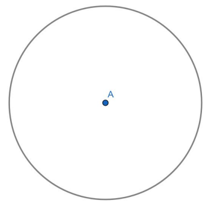
Lines AB, CD, EF are **concurrent** lines if there is a common point that lies on all of the lines AB, CD, EF. In general, lines $A_1B_1, A_2B_2 ... A_nB_n$ are concurrent if there is a point that lies on all of these lines.

Lines AB and CD are **coplanar** if they lie on the same plane.

Lines AB and CD are **parallel** if no point lies on AB and CD, and if AB and CD are coplanar. This is denoted as $AB \parallel CD$.

Have lines AB and CD intersect at X. Lines AB and CD are **perpendicular** if $\angle AXC = 90^\circ$. This is denoted as $AB \perp CD$.

Then, we shall define some shapes. We begin by defining a **circle** as the locus of points a constant distance, known as the radius, away from a point, known as the **center**.



Then, we define **triangle** ABC as the plane bounded by the lines AB, BC, CA. We can then define **quadrilateral** $ABCD$ as the plane bounded by the lines AB, BC, CD, DA. In general, for $n-gon$ $A_1A_2...A_n$, we define it as the plane bounded by the lines $A_1A_2, A_2A_3...A_{n-1}A_n, A_nA_1$.

We define $\triangle ABC$ and $\triangle DEF$ to be **congruent** if $\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$, $\overline{CA} = \overline{FD}$, $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$. This is denoted as $\triangle ABC \cong \triangle DEF$. Note that $\triangle ABC \cong DEF$ does not necessarily mean $\triangle ABC \cong \triangle EFD$.

In general, n-gons $A_1A_2...A_n$ and $B_1B_2...B_n$ are **congruent** if $\overline{A_iA_{i+1}} = \overline{B_iB_{i+1}}$ for $0 < i < n$, $\overline{A_nA_1} = \overline{B_nB_1}$, and $\angle A_j = \angle B_j$ for all $0 < j < n+1$. While this book will denote this as $A_1A_2...A_n \cong B_1B_2...B_n$, this is not a very common notation and to the best of my knowledge, this notation is unique to *Exploring Euclidean Geometry*.

Additionally, $\triangle ABC$ is **similar** to $\triangle DEF$ if there is some constant x such that $\overline{AB} = x\overline{DE}$, $\overline{BC} = x\overline{EF}$, $\overline{CA} = x\overline{FD}$, $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$. This is denoted as $\triangle ABC \sim \triangle DEF$. Similar to congruence, note that $\triangle ABC \sim \triangle DEF$ does not necessarily mean $\triangle ABC \sim \triangle EFD$.

In general, n-gons $A_1A_2...A_n$ and $B_1B_2...B_n$ are **similar** if there exists a constant x such that $\overline{A_iA_{i+1}} = x\overline{B_iB_{i+1}}$ for $0 < i < n$, $\overline{A_nA_1} = x\overline{B_nB_1}$, and $\angle A_j = \angle B_j$ for all $0 < j < n+1$. This will be denoted as $A_1A_2...A_n \sim B_1B_2...B_n$.