1. Given a triangle with side lengths 3, 4, 5, find the radius of its incircle. (1)

2. Consider acute $\triangle ABC$ with $AB = 6$, $AC = 8$, and $BC = 9$. Find the sum of all of its altitudes. (2)

3. Consider $\triangle ABC$ with integer side lengths $a, b, c$ such that $AB = c$, $AC = b$, and $BC = a$. The inradius is 2, and $ab \cdot \sin(C) = 48$. Find the side lengths of the triangle. (3)

4. A triangle has side lengths 4 and 8, and it has an area of $\sqrt{15}$. Find the length of the third side. (3)

5. A circle with area $9\pi$ is inscribed within $\triangle ABC$, and a circle with area $72.25\pi$ intersects all of the vertices of $\triangle ABC$. Provided that $\triangle ABC$ has an area of 60, find its side lengths. (2)

6. A semicircle is inscribed within a right triangle with an area of 30 such that its diameter lies on a leg of the triangle and its area is maximized. Provided that the hypotenuse of the triangle is 13, find the area of the semicircle. (3)

7. Prove that for parallelogram $ABCD$ with the lengths of $AB$ and $BC$ fixed, that $[ABCD]$ is maximized when $ABCD$ is a rectangle. (4)

8. If two side lengths of a triangle are given to be 10 and 11, what is the maximum possible area of this triangle? (★ 5)

9. In the diagram, relative lengths of some line segments are as follows:

$CE = AE$
$DB = 2AD$
$CF = 3BF$

If the area of $\triangle ABC$ is 24, what is the area of $\triangle DEF$? (3)
10. Prove that \( [ABC] = \frac{a^2 \sin B \sin C}{2 \sin A} \). (3)

11. Point \( O \) is the center of the circle circumscribed about isosceles \( \triangle ABC \). If \( AB = AC = 7 \) and \( BC = 2 \), find \( AO \). (2)

12. Given right \( \triangle ABC \), with \( AB \) as hypotenuse, prove that \( 2r = a + b - c \) where \( c \) denotes hypotenuse length, and \( r \) denotes inradius. (3)

13. Given that \( [ABC] = x \) and \( abc = y \) for \( \triangle ABC \), find \( \sin(A) \sin(B) \sin(C) \) in terms of \( x, y \). (4)