Goal: 26
Total: 38

1. Given a triangle with side lengths $3, 4, 5,$ find the radius of its incircle. (1)

2. Consider acute $\triangle ABC$ with $\overline{AB} = 6,$ $\overline{AC} = 8,$ and $\overline{BC} = 9.$ Find the sum of all of its altitudes. (2)

3. Consider $\triangle ABC$ with integer side lengths a, b, c such that $\overline{AB} = c,$ $\overline{AC} = b,$ and $\overline{BC} = a.$ The inradius is $2,$ and $ab \cdot \sin(C) = 48.$ Find the side lengths of the triangle. (3)

4. A triangle has side lengths 4 and $8,$ and it has an area of $\sqrt{15}.$ Find the length of the third side. (3)

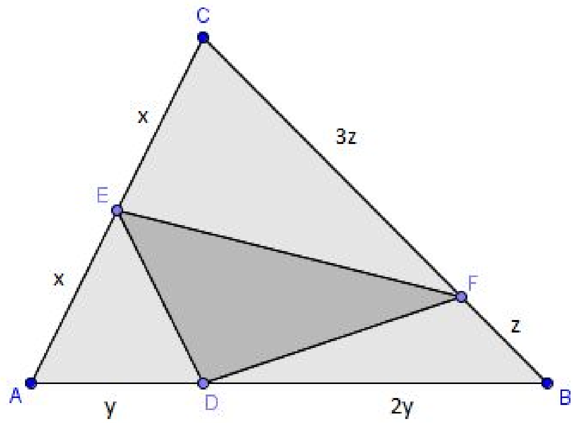
5. A circle with area 9π is inscribed within $\triangle ABC,$ and a circle with area 72.25π intersects all of the vertices of $\triangle ABC.$ Provided that $\triangle ABC$ has an area of $60,$ find its side lengths. (2)

6. A semicircle is inscribed within a right triangle with an area of 30 such that its diameter lies on a leg of the triangle and its area is maximized. Provided that the hypotenuse of the triangle is $13,$ find the area of the semicircle. (3)

7. Prove that for parallelogram $ABCD$ with the lengths of AB and BC fixed, that $[ABCD]$ is maximized when $ABCD$ is a rectangle. (4)

8. If two side lengths of a triangle are given to be 10 and 11, what is the maximum possible area of this triangle? $(\star\ 5)$

9. In the diagram, relative lengths of some line segments are as follows:
$CE = AE$
$DB = 2AD$
$CF = 3BF$
If the area of $\triangle ABC$ is $24,$ what is the area of $\triangle DEF$? (3)



10. Prove that $[ABC] = \frac{a^2 \sin B \sin C}{2 \sin A}$. (3)

11. Point O is the center of the circle circumscribed about isosceles $\triangle ABC$. If $AB = AC = 7$ and $BC = 2$, find AO. (2)

12. Given right $\triangle ABC$, with AB as hypotenuse, prove that $2r = a + b - c$ where c denotes hypotenuse length, and r denotes inradius. (3)

13. Given that $[ABC] = x$ and $abc = y$ for $\triangle ABC$, find $\sin(A)\sin(B)\sin(C)$ in terms of x, y. (4)