Definition

Triangles have three sides, three vertices, and three angles. Area refers to how much space a two-dimensional figure takes up. Area of a Triangle is important because all polygons can be split into triangles.

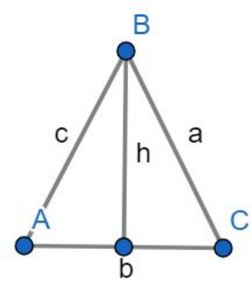
Notation

Take $\triangle ABC$ and denote BC, AC, AB as a, b, c and $\angle BAC, \angle ABC, \angle ACB$ as $\angle A, \angle B, \angle C,$ respectively.

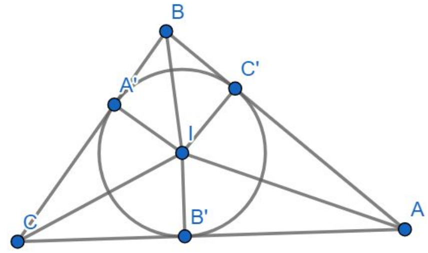
Formulas

$\frac{bh}{2}$ states that if you drop an altitude from B and let it intersect AC at $X,$ the area of $\triangle ABC$ is $\frac{bh}{2}$ where h denotes the height of the altitude, or $BX.$ You already know the proof for this one.

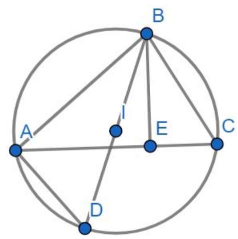
$\frac{1}{2}ab\sin C$ states that given two sides of a triangle a, b and the angle C between them, the area of said triangle is $\frac{1}{2}ab\sin C.$ The proof for this one relies on the definition of sine; remember to drop an altitude such that it is the opposite of $C.$



$[ABC] = rs$ states the area of a triangle is the product of its inradius and semiperimeter. Splitting the initial triangle into three smaller triangles using the incenter and noting a tangent is perpendicular to the radius gives us the result.



$[ABC] = \frac{abc}{4R}$ uses the circumradius and the product of the sides of the triangle. A combination of similarity and $\frac{bh}{2}$ yields the result.



Heron's Formula states that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$ where s denotes semiperimeter. The proof for this involves either Pythagorean or Law of Cosines coupled with $\frac{bh}{2}$.